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Thermodynamical disturbances in a nonlocal thermoelastic medium under Lord-Shulman model

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Abstract

The objective of this manuscript is to examine the thermodynamical disturbances in a nonlocal isotropic thermoelastic solid half-space. The enunciation is applied to Lord-Shulman model of generalized thermoelasticity with Eringen's nonlocal elasticity theory. The formulation is subjected to a mechanical load. The expressions for the displacement components, stresses and temperatureare obtained by using normal mode technique and the numerical computations have been carried out with the help of MATLAB software. The comparisons are made among the results obtained by taking into account the different values of nonlocal parameter ($\varepsilon = 0.195 \times 10^{-9}, 0.195 \times 10^{-4}, 0.195 \times 10^{-1}$) and time (t = 0.1, 0.3, 0.5). The outcomes point out a strong impact of the nonlocality and time on the physical quantities and agree with the boundary conditions.

Keywords: Lord-Shulman model; Nonlocal thermoelasticity; Mechanical load; Normal mode analysis.

1. Introduction

The generalized thermoelasticity theories involve hyperbolic-type heat conduction equations which admit finite speed of thermal signals. Lord and Shulman [1] developed the first generalized theory of thermoelasticity by modifying the Fourier's law of heat conduction, in which one relaxation time was introduced. It is also referred as LS theory of generalized thermoelasticity. Later on, Green and Lindsay [2] developed another theory of generalized thermoelasticity by including two different relaxation times in the constitutive relations. This theory is also known as temperature rate dependent thermoelasticity and referred as GL theory. A lot of research work has been carried out based on these theories. For this purpose, one can refer to Othman et al [3,] Ezzat and Youssef [4], Kumar et al. [5], Zenkour and Abbas [6], Abbas and Kumar [7], Hobiny and Abbas [8,9] etc.

Eringen and Edelen [10] and Eringen [11,12] extended the concept of nonlocality to elasticity and developed the theory of nonlocal elasticity. In local elasticity, the stress at a point isa function of strain at that point only, whereas in nonlocal elasticity the stress at any point is a function of strain at all other points of the medium. This indicates that the nonlocal stress forces act as remote action forces. These types of forces are frequently encountered in atomic theory of lattice dynamics. Balta and Suhubi [13] developed another theory of nonlocal thermoelasticity. In this theory, the entropy inequality was used in classical form. By using the concept of nonlocal elasticity, Acharya and Mondal [14] discussed the propagation of Rayleigh surface waves in a viscoelastic solid. Roy et al. [15] examined the combined effect of magnetic field and rotation on nonlocal Rayleigh surface waves. Challamel et al. [16] studied a nonlocal generalization of the heat equation that can be based on lattice arguments. The applications of the nonlocal thermoelasticity theories have been examined extensively Khurana and Tomar [17].

The main objective of the present investigation is to study the thermo-mechanical interactions in a nonlocal isotropic material due to the application of mechanical load. Exact expressions for different field quantities are carried out by using normal mode technique. The numerical results are also calculated and illustrated graphically. to estimate and highlight the impact of different parameters like nonlocal parameter and time. The novelty of the present study resides in the fact that we have proposed quite new research to study the dependence of various field quantities on nonlocal parameter and time for an isotropic medium based on LS model with Eringen's nonlocal elasticity theory.

2. Derivation of fundamental equations

The constitutive relations for a nonlocal, isotropic thermoelastic solid are given as:

$$(1-\varepsilon^2\nabla^2)\sigma_{ij} = \sigma_{ij}^L = 2\mu e_{ij} - (\lambda e - \beta\theta)\delta_{ij}, \tag{1}$$

$$(1-\varepsilon^2\nabla^2)\rho T_0 S = (\rho T_0 S)^L = \beta_{ij} T_0 e + \rho C_E \theta, \tag{2}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{3}$$

The equation of motion for an isotropic, nonlocal thermoelastic material in the absence of body forces and heat sources is given by:

$$\sigma_{ji,j} = \rho \ddot{u}_i. \tag{4}$$

The nonlocal generalization of Fourier's law for thermoelastic solid can be described as:

$$(1-\varepsilon^2\nabla^2)(q_i+\tau_0\dot{q}_i)=K\theta_{,i}, \qquad (5)$$

In the context of linear theory of thermoelasticity, the energy equation has the form

$$\rho T_0\dot{S}=q_{i,i}. \qquad (6)$$

By virtue of (2), (5) and (6), the heat conduction equation takes the form

$$K\theta,ii=\left(1+\tau_0\frac{\partial}{\partial t}\right)\left(\beta T_0\dot{e}+\rho C_E\dot{\theta}\right). \qquad (7)$$

where u_i are the components of displacement vector $\vec{u},\theta=T-T_0,T$ is the absolute temperature, T_0 is reference temperature assumed to obey the inequality $|\theta/T_0|\ll 1,\sigma_{ij}$ are the components of the stress tensor, e_{ij} are the components of strain tensor, δ_{ij} is the Kronecker delta function, e is the cubical dilatation, ρ is the density of the medium, C_E is the specific heat, K is the thermal conductivity, $\beta=(3\lambda+2\mu)\alpha_t,\alpha_t$ is the coefficient of linear thermal expansion, λ and μ are the Lame's constants, $\varepsilon=e_0a_{cl}$ is the nonlocal parameter, a_{cl} denotes the internal characteristic length and e_0 is a material constant and the quantity σ_{ij}^L correspond to the classical local thermoelastic solid, τ_0 is the relaxation time. In the above relations, a superposed dot indicates derivative with respect to time and a comma represents derivative with respect to spatial variable.

3. Problem formulation

We choose the rectangular cartesian coordinates system (x,y,z) having the surface of the halfspace as the plane $z=0$, with z-axis pointing vertically downwards into the medium.

In xz-plane, the displacement components are taken as:

$$u=u(x,z,t),v=0,w=w(x,z,t). \qquad (8)$$

In view of expression (8), the stresses arising from Eq. (1) can be written as:

$$(1-\varepsilon^2\nabla^2)\sigma_{xx}=\sigma_{xx}^L=(\lambda+2\mu)\frac{\partial u}{\partial x}+\lambda\frac{\partial w}{\partial z}-\beta\theta, \qquad (9)$$

$$(1-\varepsilon^2\nabla^2)\sigma_{zz}=\sigma_{zz}^L=\lambda\frac{\partial u}{\partial x}+(\lambda+2\mu)\frac{\partial w}{\partial z}-\beta\theta, \qquad (10)$$

$$(1-\varepsilon^2\nabla^2)\sigma_{zx}=\sigma_{zx}^L=\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right). \qquad (11)$$

Inserting the components of stresses defined in Eqs. (9) - (11), into Eq. (4), we obtain

$$\rho(1-\varepsilon^2\nabla^2)\frac{\partial^2 u}{\partial t^2}=(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}+\mu\frac{\partial^2 u}{\partial z^2}+(\lambda+\mu)\frac{\partial^2 w}{\partial x\,\partial z}-\beta\frac{\partial\theta}{\partial x}, \qquad (12)$$

$$\rho(1-\varepsilon^2\nabla^2)\frac{\partial^2 w}{\partial t^2} = \mu\frac{\partial^2 w}{\partial x^2} + (\lambda+2\mu)\frac{\partial^2 w}{\partial z^2} + (\lambda+\mu)\frac{\partial^2 u}{\partial x\,\partial z} - \beta\frac{\partial \theta}{\partial z}. \quad (13)$$

Using summation convention, heat conduction equation (7) takes the form

$$K\nabla^2\theta = \left(1+\tau_0\frac{\partial}{\partial t}\right)\left[T_0\beta\dot{e} + \rho C_E\frac{\partial\theta}{\partial t}\right]. \quad (14)$$

For convenience, we will make use of the following non-dimensional variables to normalize the above relations

$$(x',z',u',w',\varepsilon') = \frac{\omega^*}{C_1}(x,z,u,w,\varepsilon),(t',\tau_0') = \omega^*(t,\tau_0),\theta' = \frac{\theta}{T_0},$$
$$\sigma_{ij}' = \frac{1}{\rho C_1^2}\sigma_{ij}, \text{ where, } \omega^* = \frac{C_E(\lambda+2\mu)}{K_1}, C_1^2 = \frac{(\lambda+2\mu)}{\rho}. \quad (15)$$

By using the Helmholtz decomposition, in the two dimensional xz-plane, the displacement components can be expressed as

$$u = \frac{\partial\phi}{\partial x} - \frac{\partial U}{\partial z}, \quad w = \frac{\partial\phi}{\partial z} + \frac{\partial U}{\partial x}. \quad (16)$$

By introducing the above dimensionless quantities and substituting the expressions (16) into Eqs. (9)-(14), makes it

$$(1-\varepsilon^2\nabla^2)\sigma_{xx} = \sigma_{xx}^L = \frac{\partial^2\phi}{\partial x^2} + A_1\frac{\partial^2\phi}{\partial z^2} + (A_1-1)\frac{\partial^2 U}{\partial x\,\partial z} - A_2\theta, \quad (17)$$

$$(1-\varepsilon^2\nabla^2)\sigma_{zz} = \sigma_{zz}^L = A_1\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} + (1-A_1)\frac{\partial^2 U}{\partial x\,\partial z} - A_2\theta, \quad (18)$$

$$(1-\varepsilon^2\nabla^2)\sigma_{zx} = \sigma_{zx}^L = A_3\left(2\frac{\partial^2\phi}{\partial x\,\partial z} + \frac{\partial^2 U}{\partial x^2} - \frac{\partial^2 U}{\partial z^2}\right), \quad (19)$$

$$(1-\varepsilon^2\nabla^2)\frac{\partial^2\phi}{\partial t^2} = \nabla^2\phi - A_2\theta, \quad (20)$$

$$(1-\varepsilon^2\nabla^2)\frac{\partial^2 U}{\partial t^2} = A_3\nabla^2 U, \quad (21)$$

$$\nabla^2\theta = \left(1+\tau_0\frac{\partial}{\partial t}\right)\left(A_4\nabla^2\frac{\partial\phi}{\partial t} + \frac{\partial\theta}{\partial t}\right). \quad (22)$$

where $A_1 = \frac{\lambda}{\rho C_1^2}, A_2 = \frac{\beta T_0}{\rho C_1^2}, A_3 = \frac{\mu}{\rho C_1^2}, A_4 = \frac{\beta C_1^2}{K\omega^*}$.

2. Solution methodology

In the present section, normal mode technique is applied to obtain the solution of the physical quantities. So, the physical variables under consideration can be decomposed in terms of normal modes in the following form

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$$\left[u, w, \phi, U, \theta, \sigma_{ij}, \sigma_{ij}^{L}\right](x, z, t) = \left[u^*, w^*, \phi^*, U^*, \theta^*, \sigma_{ij}^*, \sigma_{ij}^{L*}\right](z) e^{(\omega t + \imath m x)}, \quad (23)$$

where $u^*, w^*, \phi^*, U^*, \theta^*$ and σ_{ij}^* are the amplitudes of the physical quantities, ω is the angular frequency, $\imath$ is the imaginary unit and m is the wave number in x-direction.

By virtue of expression (23), Eqs. (20) $-$(22) turn to the following forms

$$(B_1 D^2 + B_2)\phi^* - A_2 \theta^* = 0, \quad (24)$$

$$(B_3 D^2 + B_4) U^* = 0, \quad (25)$$

$$(B_5 D^2 + B_6)\phi^* + (D^2 + B_7)\theta^* = 0, \quad (26)$$

where $D = \frac{\partial}{\partial z}, B_1 = (1 + \varepsilon^2 \omega^2), B_2 = -(m^2 + (1 + \varepsilon^2 m^2)\omega^2), B_3 = (A_3 + \omega^2 \varepsilon^2),$

$$B_4 = -(A_3 m^2 + (1 + m^2 \varepsilon^2)\omega^2), B_5 = -A_4 \omega (1 + \tau_0 \omega), B_6 = -B_5 m^2 B_7 = -(m^2 + (1 + \tau_0 \omega)\omega)$$

Now, solving Eqs. (24) and (26) simultaneously, we get fourth-order ordinary differential equation satisfied by $\phi^*(z)$ and $\theta^*(z)$ as

$$(I_1 D^4 + I_2 D^2 + I_3)[\phi^*(z), \theta^*(z)] = 0, \quad (27)$$

where $I_1 = B_1, I_2 = B_1 B_7 + B_2 + A_2 B_5, I_3 = B_2 B_7 + A_2 B_6.$

The solutions of Eqs. (25) and (27) under the assumption that these are bounded at infinity, can be expressed as

$$U^*(z) = M_3(m, \omega 1) e^{-\lambda_3 z}, \quad (28)$$

$$[\phi^*, \theta^*](z) = \sum_{i=1}^{2} [1, H_{1i}] M_i(m, \omega) e^{-\lambda_i z}, \quad (29)$$

where$H_{1i} = \frac{(B_1 \lambda_i^2 + B_2)}{A_2}, \lambda_i^2 = \frac{-I_2 \pm \sqrt{(I_2)^2 - 4 I_1 I_3}}{2 I_1} (i = 1,2), \lambda_3^2 = \frac{-B_4}{B_3}.$

Substituting Eqs. (28) and (29) in Eqs. (16)-(19), the expressions for displacement and stress components can be deduced as

$$[\sigma_{zx}^{L*}, \sigma_{zz}^{L*}, u^*, w^*](z) = \sum_{i=1}^{3} [H_{2i}, H_{3i}, H_{4i}, H_{5i}] M_i(m, \omega) e^{-\lambda_i z}, \quad (30)$$

where $H_{2i} = -2 i m \lambda_i A_3, H_{23} = -A_3(m^2 + \lambda_3^2), H_{3i} = (\lambda_i^2 - m^2 A_1 - A_2 H_{1i}),$

$$H_{33} = -\imath m \lambda_3 (1 - A_1), H_{4i} = \imath m, H_{43} = \lambda_3, H_{5i} = -\lambda_i, H_{53} = \imath m, \quad i = 1,2.$$

3. Application: Mechanical load on the surface of half-space

4. Mechanical boundary conditions:

The boundary plane $z = 0$ is subjected to a mechanical load. So, the mechanical boundary conditions for a nonlocal generalized thermoelastic medium are given by:

$$\sigma_{zz}(x, 0, t) = -f(x, t), \quad \sigma_{zx}(x, 0, t) = 0. \tag{31}$$

As a simplification, we assume that the boundary conditions (31) are equivalent to local boundary conditions, which are given by:

$$\sigma_{zz}^{L}(x, 0, t) = -f(x, t), \quad \sigma_{zx}^{L}(x, 0, t) = 0, \tag{32}$$

where $f(x, t)$ is a given function of x and t.

5. Thermal boundary condition:

The thermal boundary condition is vanishing of temperature θ i.e.

$$\theta(x, 0, t) = 0. \tag{33}$$

Taking into account the non-dimensional expressions for temperature and stresses from (29) and (30) with $f' = \frac{f(x,t)}{\rho C_1^2}$ and applying normal mode technique defined in expression (23), the above boundary conditions reduce to a non-homogeneous system of three equations, which can be written in matrix form as follows:

$$\begin{bmatrix} H_{31} & H_{32} & H_{33} \\ H_{21} & H_{22} & H_{23} \\ H_{11} & H_{12} & 0 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} -f^* \\ 0 \\ 0 \end{bmatrix}, \tag{34}$$

where f^* is the magnitude of the mechanical load.

On solving the system of Eq. (34), one can obtain $M_i (i = 1,2,3)$ as:

$$M_1 = \frac{\Delta_1}{\Delta}, M_2 = \frac{\Delta_2}{\Delta}, M_3 = \frac{\Delta_3}{\Delta}, \tag{35}$$

where $\Delta = H_{31}L_1 - H_{32}L_2 + H_{33}L_3, \Delta_1 = -f^*L_1, \Delta_2 = f^*L_2, \Delta_3 = -f^*L_3,$
$L_1 = -H_{23}H_{12}, L_2 = -H_{11}H_{23}, L_3 = H_{21}H_{12} - H_{11}H_{22}.$

Substituting (35) into expressions (28) and (30), we obtain the expressions for displacement components, stresses and temperature distribution as:

$$\theta^*(z) = \frac{1}{\Delta} \sum_{i=1}^{2} H_{1i} \Delta_i e^{-\lambda_i z}, \tag{36}$$

$$[\sigma_{zz}^{L*}, \sigma_{zx}^{L*}, u^*, w^*](z) = \frac{1}{\Delta}\sum_{i=1}^{3} (H_{2i}, H_{3i}, H_{4i}, H_{5i})\Delta_i e^{-\lambda_i z}. \qquad (37)$$

6. Computational results and discussion

In order to study the effects of nonlocal parameter and time on the field variables, a numerical analysis is carried out. For the purpose of illustration, we have chosen a zinc crystal-like material. The material constants are taken as (Eringen [14]):

$$\lambda = 3.62\times10^{10}\mathrm{Nm^{-2}}, \mu = 3.85\times10^{10}\mathrm{Nm^{-2}}, \beta = 5.75\times10^{6}\mathrm{Nm^{-2}K^{-1}}, T_0 = 296K$$
$$K = 1.24\times10^{2}\mathrm{Wm^{-1}K^{-1}}, \tau_0 = 0.02\mathrm{s}, \rho = 7.14\times10^{3}\mathrm{kgm^{-3}}, C_E = 0.39\times10^{3}\mathrm{Jkg^{-1}K^{-1}}$$
$$a = 0.5\times10^{-9}\mathrm{m}, e_0 = 0.39.$$

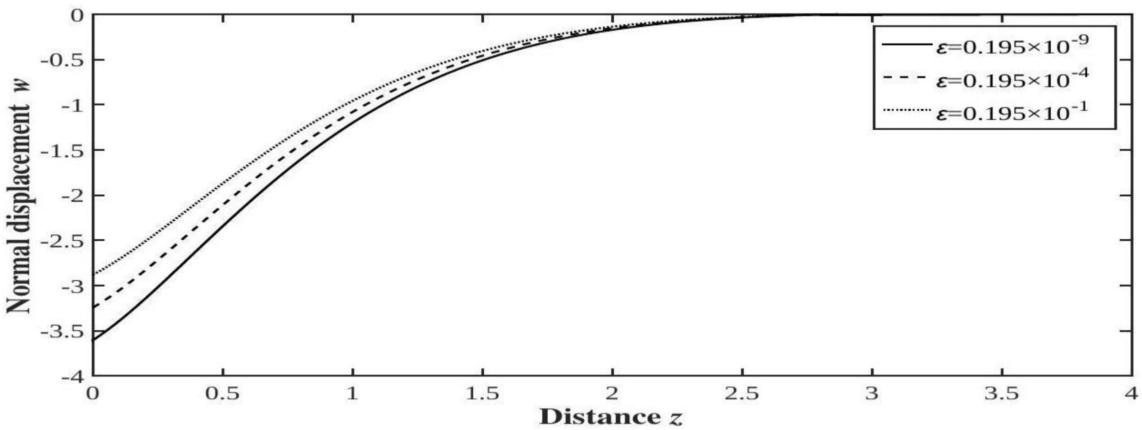

Figure 1: Effect of nonlocal parameter on normal displacement distribution

Group I: In Figures1-4, the variations of field variables are illustrated for three values of nonlocal parameter ($\varepsilon = 0.195 \times 10^{-9}$, 0.195×10^{-4}, 0.195×10^{-1}). Figure 1 reveals the variations of normal displacement w with distance $z$for three different values of nonlocal parameter. It is clear from the plot that with an increase in value of nonlocal parameter, there is a decrease in the numerical values of normal displacement, which shows that nonlocal parameter ε has a decreasing effect on the normal displacement.Figure 2 illuminates the variations of normal stress component σ_{zz}^{L} with distance z for three different values of nonlocal parameter. From the plot, it is clear that nonlocal parameter has a prominent decreasing effect on the profile of normal stress.

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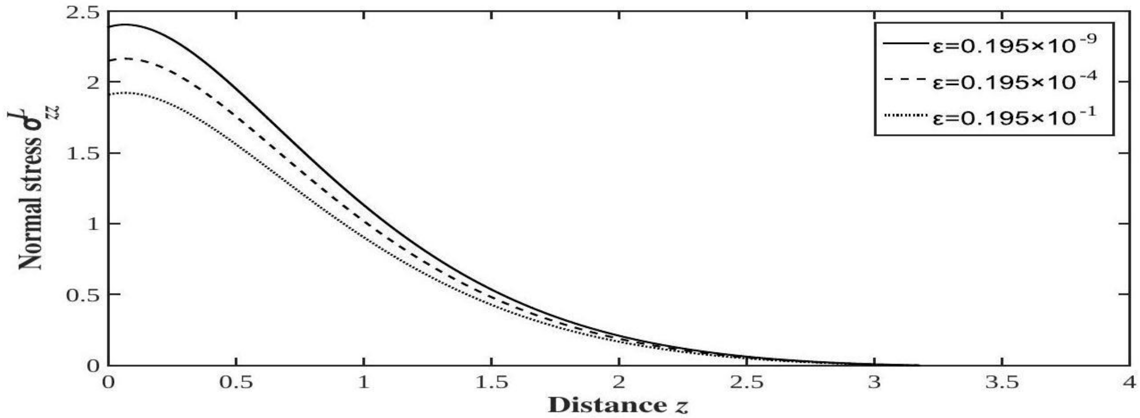

Figure 2: Effect of nonlocal parameter on normal stress distribution

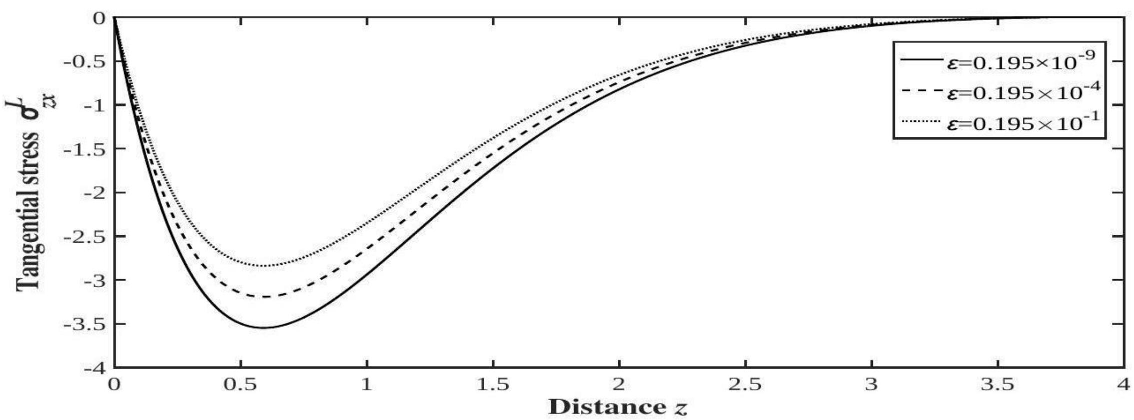

Figure 3: Effect of nonlocal parameter on tangential stress distribution

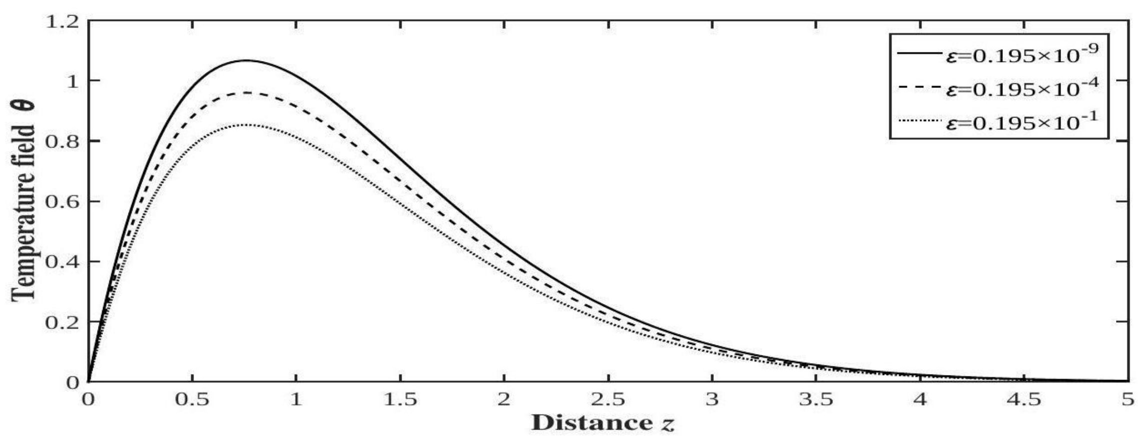

Figure 4: Effect of nonlocal parameter on temperature distribution

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Figure 3 shows the profile of tangential stress σ_{zx}^{L} with distance z for different values of nonlocal parameter. Tangential stress starts with a zero value for all the curves, which is in quite good agreement with the boundary condition. It can be seen from the figure that nonlocal parameter has a decreasing effect on tangential stress. The pattern of variation of temperature field θ for different values of nonlocal parameter ε has been shown in Figure 4. The temperature field is having a similar pattern of distribution in all the three cases. As we increase the value of nonlocal parameter, the numerical values of temperature field decreases.

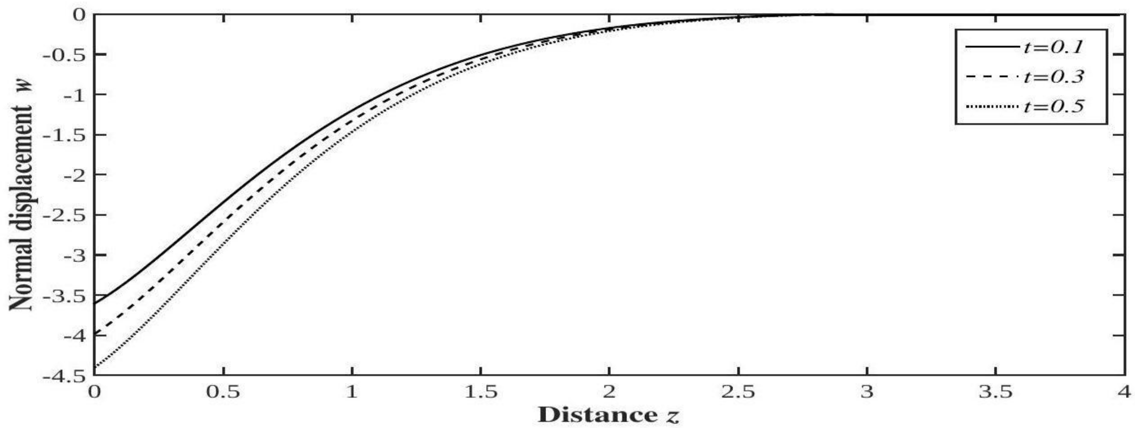

Figure 5: Effect of time on normal displacement distribution

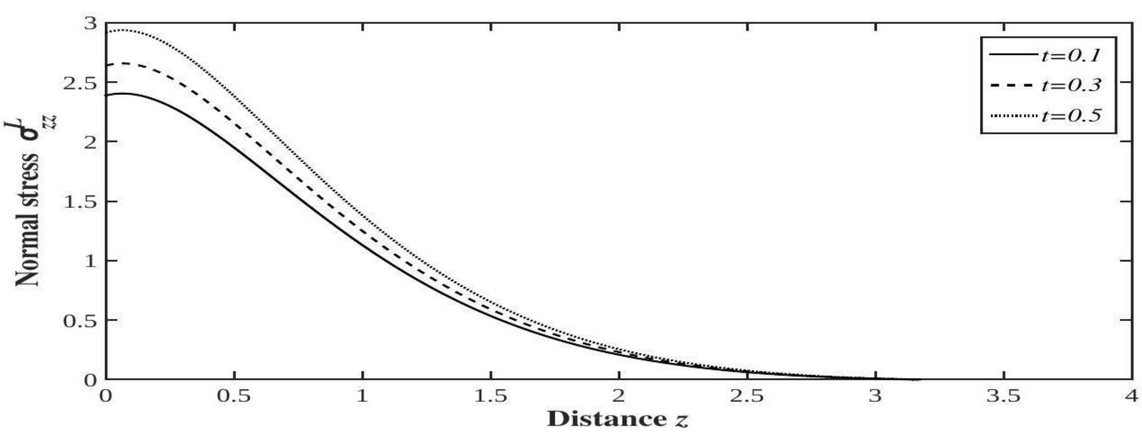

Figure 6: Effect of time on normal stress distribution

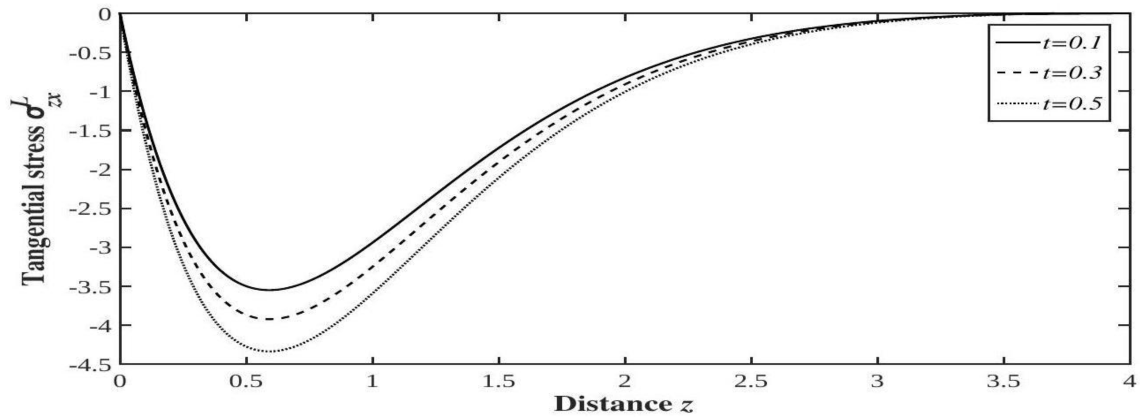

Figure 7: Effect of time on tangential stress distribution

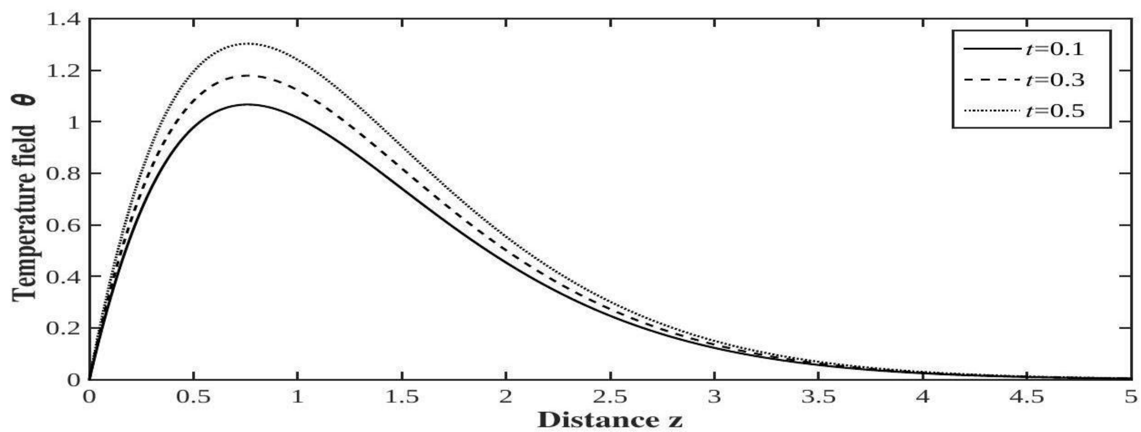

Figure 8: Effect of time on temperature distribution

Group II: In Figures 5-8, we have explored the effects of time t taken to be t = 0.1, 0.3, 0.5 on the spatial variations of physical fields. In Figure 5, a graphical representation is given for variations of normal displacement w with distance z for three different values of time. An increment in the value of time results in an increment in the numerical values of the normal displacement, which elucidates the fact that time is having a noticeable increasing effect on the profile of normal displacement. Figure 6 displays the profile of normal stress versus distance z for three different values of time. Time has a spectacular increasing effect on the profile of normal stress and the effect disappears as we move away from the point of application of the source. In Figure 7, we have depicted the behavior of tangential stress with distance z for three different values of time. Time has a prominent increasing effect on the profile of tangential stress. The impact of time on temperature field θ is demonstrated in Figure 8. The behavior of temperature field for all the values of time is almost similar. The numerical value of temperature field increases as the value of time increases, which shows that time has an increasing effect on temperature field.

9. Concluding remarks

The main goal of present investigation is to provide a mathematical model for determining the behavior of normal displacement, normal stress, tangential stress and temperature in a nonlocal isotropic thermoelastic medium under LS theory by using normal mode technique. From this research, one can infer that

- From all the figures, it is clear that all the physical variables have non-zero values only in the limited domain of space, which is in accordance with the notion of generalized thermoelasticity theory and supports all the physical facts.
- It is noticed from the figures that all the physical field quantities satisfy the boundary conditions.
- All the field variables are found to be very sensitive towards the nonlocal parameter. The increase in the value of nonlocal parameter acts to decrease the magnitudes of all the physical quantities.
- All the physical quantities show similar pattern for different values of time and an increment in the value of time causes an increment in the magnitude of all the field variables.

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